

On T.R.A.C.K.S.

Teaching Resource Activities and Conservation to Kansas Students

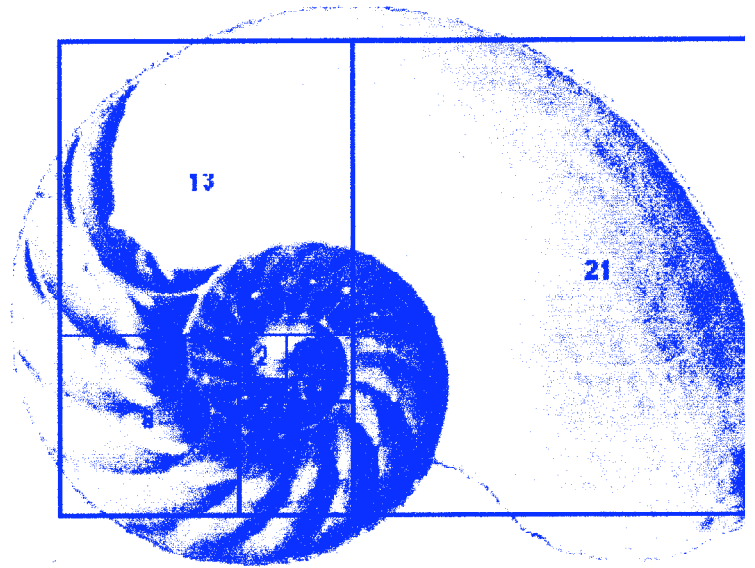


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Nature's Math



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Don't Miss
Our Next
Issue:

Wildlife
Mythbusters

Nature's Math

Welcome to this issue of On Tracks devoted to Nature's Math. The activities included in this issue cover spider webs, trees, and patterns in nature. These activities will address the math standards for the state of Kansas, for the most part, within the 5th through 8th grade, with some activities even being appropriate for high school. These activities may also be modified for younger students, especially Geometry of Spider Webs.

Math Standard Benchmark	Standard 1 Numbers & Computation				Standard 2 Algebra				Standard 3 Geometry				Standard 4 Data	
	1	2	3	4	1	2	3	4	1	2	3	4	1	2
Geometry of Spider Webs									X	X	X			
Measuring Trees	X	X	X	X					X		X			
Patterns in Nature	X	X	X	X	X	X	X	X					X	X



Geometry of Spider Webs

Objective: Students will construct their own spider web using the principles of geometry.

Background: All spiders of the same species build the same webs, but spiders from different species build different webs. Web building is **instinctive** for spiders. Baby spiders hatch knowing how to design their webs. No one has to teach them how to do it.

The web of a spider can appear very complex in its design and construction, however, observations reveal that the common garden spider constructs its web by following a few relatively simple rules. There are three parts or components of the web:

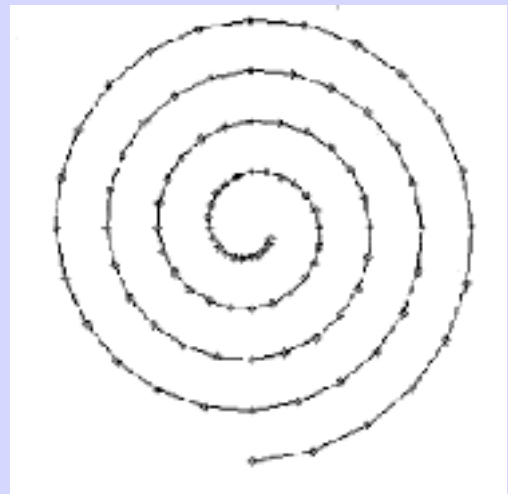
1. **radial threads**, which converge in a central spot called the hub. The radial threads are like the spokes of a bicycle wheel
2. **frame threads**, which go around the outside of the web and serve as attachment sites for the radial threads.
3. The **capture spiral**.



The basic supporting structure of the orb web is provided by the radial and frame threads, neither of which is sticky. The catching spiral, in contrast, consists of a thread studded with glue droplets.

An Arithmetic Spiral

When laying down the sticky capture spiral, the spider starts at the outside edge, always going in one direction, and winding inward towards the hub. At each crossing of a radial thread, the spider fastens the sticky thread to the radius. Both front legs and hind legs play an important role in this process. One front leg reaches for the radius and touches it with a quick, brushing movement to locate its position. At the same time, one of the fourth legs pulls the thread out of the spinnerets and dabs it against the radius. By doing so, the spider keeps the distance between successive capture spiral turns about equal. **The geometry of the capture spiral is therefore equidistant or arithmetic.**



Materials Needed:

1 skein of thread
1 thin wire clothes hanger
scissors
magic marker
a small piece of cardboard 1" x 2"

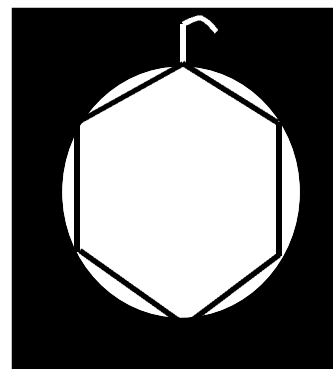
Procedure:

Step 1: Take the wire coat hanger and shape the bottom triangular portion into a circle.

Step 2: With a magic marker, make a mark on the hanger at the bottom opposite the hooked part used for hanging. Then divide each half of the circle into approximately three equal sections. You should have six sections marked.

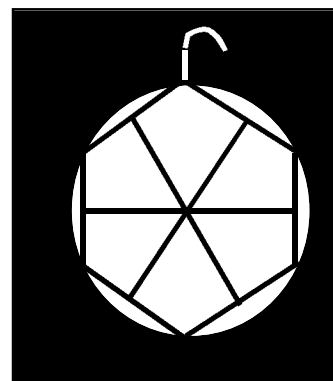
Step 3: Cut a 4' piece of thread. Starting at the top under the hook on the right side, attach a thread to the wire hanger with a double knot. Moving to the left, attach the thread with a double knot to the left side as well. These knots will only be about 1/2" apart.

Step 4: Still moving to the left, take the same thread and make a knot at the first mark on your hanger. (This line now makes a **chord** of the circle) Continue around the circle making a knot at each mark around the circle until you reach the top starting point. Keep your thread fairly tight as you work your way around the circle. Make a double knot at the top to secure the thread and cut off any extra. Your hanger should be stretched with six segments of thread like the example shown.

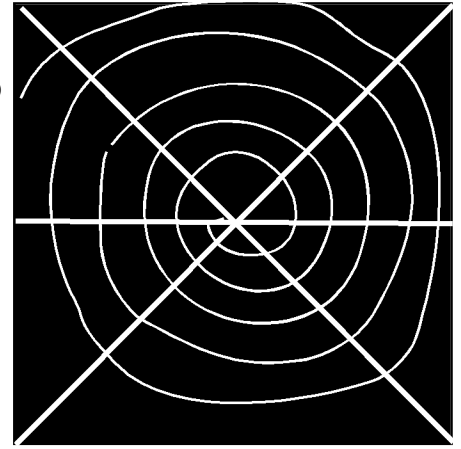


Step 5: Take 10-15 ft of thread from the skein and wrap it around the cardboard piece to make it easier to work with.

Step 6: Cut a new piece of thread about 1 foot long and attach it with a knot to the middle section of the top left thread. Stretch it diagonally to the opposite side and knot it in the center of that thread too. Repeat the process 2 more times, attaching a thread in the middle of each segment and stretching it diagonally to the opposite side until you have the center crisscrossed with threads as shown. Trim thread ends as needed.

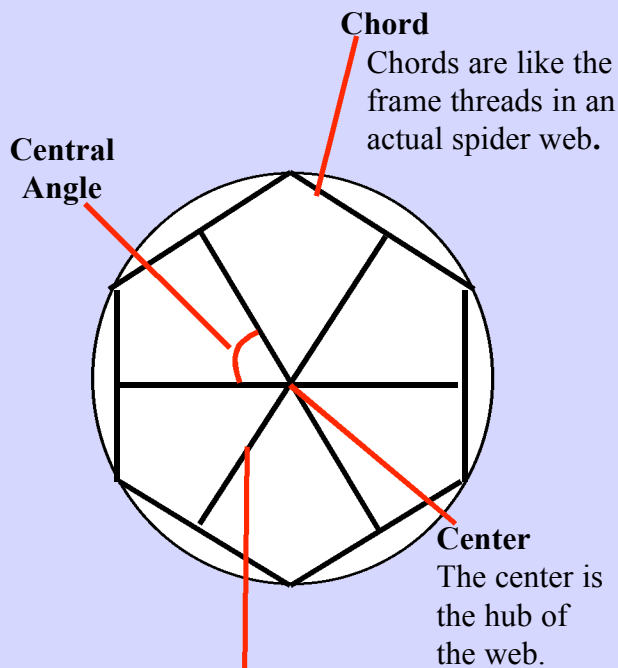


Step 7: Using the thread you wrapped around the cardboard and starting in the middle of the circle (where all the threads crisscross), secure your new thread with a double knot to keep all the threads in the center together. When this is done, take your thread and weave it in a circle, going from left to right, making a left to right knot on each thread as you come to it. *(A left to right knot is like the first step in tying your shoes. Take the string and go over and under the radial string, then pull gently to tighten)* Keep your thread fairly tight as you do this. Keep following this same procedure as you make ever-widening rotations around the circle. Leave about 1/2" to 3/4" between each successive row of circles that you are creating. Do this for as many rotations as it takes to reach the outer edge of the circle. This should give you 8-12 rows, depending on how far apart each rotation is made. If you run out of thread while you are doing this, simply knot more string to the old and continue.



Step 8: When you get to the top (under the hook of the hanger) on your last rotation, secure your thread with a double knot and trim any excess thread. Now you have a spider web!

What parts of a circle do you recognize in your web?



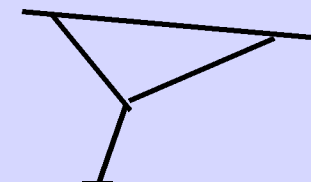
This line bisects the chord at a right angle. It acts as a radial thread of the spider web

How a REAL spider web is constructed

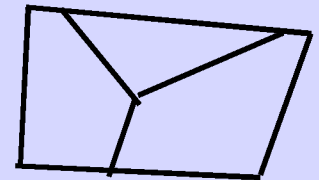
1. The first thread is a sticky thread released by the spider and carried by the wind until it sticks to something. Next, the spider reinforces this bridge.



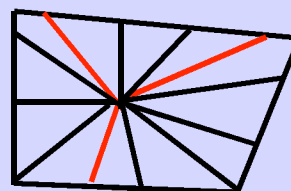
2. The spider constructs a loose thread.



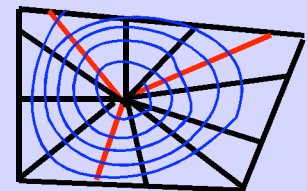
3. The spider turns the loose thread into a "Y" shape forming three radii



4. The spider constructs the frame around the three radii



5. More radii are added.



6. The capture spiral is woven between the radii



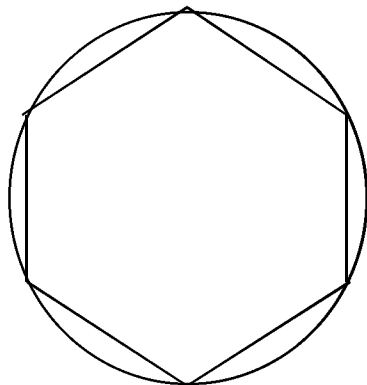
Geometry of Spider Webs Worksheet

1. Using your spider web and frame as an example, how many chords of your circle (clothes hanger) did you create?

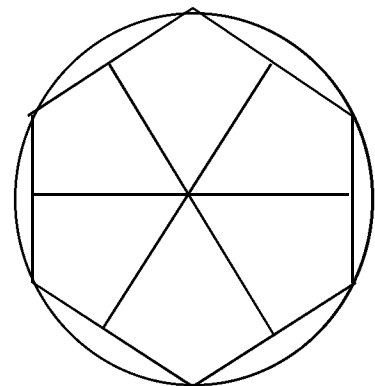
2. What is the polygon created by the chords?

3. Is the thread that bisects the chord equal to the diameter of the circle?
Why or Why not?

4. Draw a line in the figure below that would equal the diameter of the circle but also connect two opposite chords?



5. Shade half of the figure to the right to show mirror symmetry. (There are several possibilities).



6. How could you prove that the figure from #5 has two congruent angles?

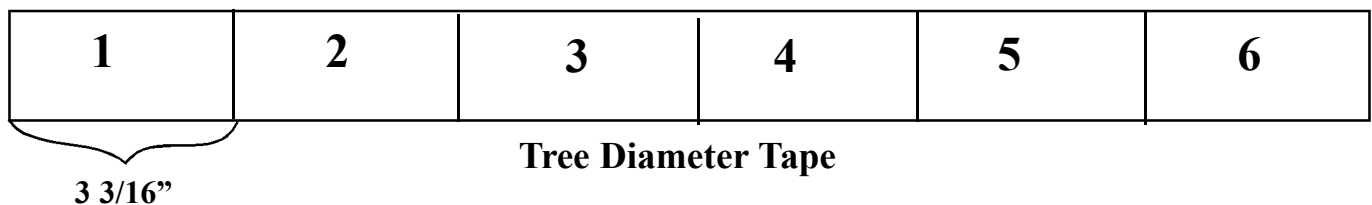


Measuring Trees

Objective: Students will measure trees in different ways and practice estimating volume called board feet.

Background: Trees, and the lumber obtained from them, are one of our most important renewable natural resources. Foresters measure trees to help plan forest management and when to cut certain trees. **By measuring a tree's diameter and height, one can estimate the volume of lumber for any standing tree.** To determine the approximate amount of timber in a stand of trees, foresters do a "timber cruise" in which they calculate the volume of lumber in a given area, examine the health of the forest, and count the number of different trees found there.

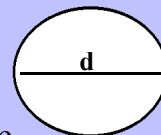
Getting Ready: Making a Tree Diameter Tape. Take heavy paper or fabric and cut a strip that is 1 1/2" wide and 45" in length. (Paper strips will need to be taped together end-to-end to reach 45") Starting at one end of your paper or fabric, mark off units 3.14 inches apart (about 3 3/16"), numbering units starting with one. This will give you a reading in inches for the tree's **diameter**. Since you can only measure the circumference (or distance around the tree), each 3.14 inches of circumference will equal one inch of diameter.



Why does measuring the circumference give me the diameter?

The circumference of a circle is the distance around the circle. It is found by the formula:

Circumference = πd d is the diameter and $\pi = 3.14$



The diameter of the tree **could be** found by measuring the distance around the tree with an ordinary tape measure and dividing that number by π or 3.14"

Example: If the circumference = 50", then the equation becomes:

$$50'' = (3.14'') d$$

$$\frac{50''}{3.14''} = d$$

$$d = 15.9''$$

$$d = 15.9''$$

By creating a tape marked off in "numbers of units of π (3.14)", we shorten the process of finding the diameter because we have already divided by π .



Procedure:

1. Find the diameter of the tree you have selected.

Foresters always measure the diameter of a tree at **4.5 feet** (1.4 m) above the ground. This measurement is called **Diameter at Breast Height** or **DBH** and is the standard or “rule” for all foresters.

Using the tree diameter tape you just made, find the diameter of the tree **4.5 ft** above the ground. This will be the number read off the tape. Record your DBH on the worksheet.

2. Determine the height of your tree.

You will determine the height of your tree using the proportional method. Here’s how:

- Have your partner stand at the base of the tree to be measured.
- Hold a ruler at arm’s length (**zero end at the top**) and walk backwards, keeping your arm stiff, until the top and bottom of the ruler line up with the top and bottom of the tree.
- Read on the ruler where the top of your partner’s head appears. (For example: at 2 inches) Record on the worksheet.
- Divide the length of the ruler by the number obtained in “step c”. (For example: 12 inches ÷ 2 inches = 6 inches). This number becomes the **proportional factor**. Record this on the worksheet.

$$\text{Proportional factor} = \frac{\text{ruler length}}{\text{ruler height of person}} \quad \frac{(a)}{(b)}$$

- Measure your partner’s actual height in inches. Record.
- Multiply your partner’s height by the proportional factor. Record.
- Calculate the height of the tree.

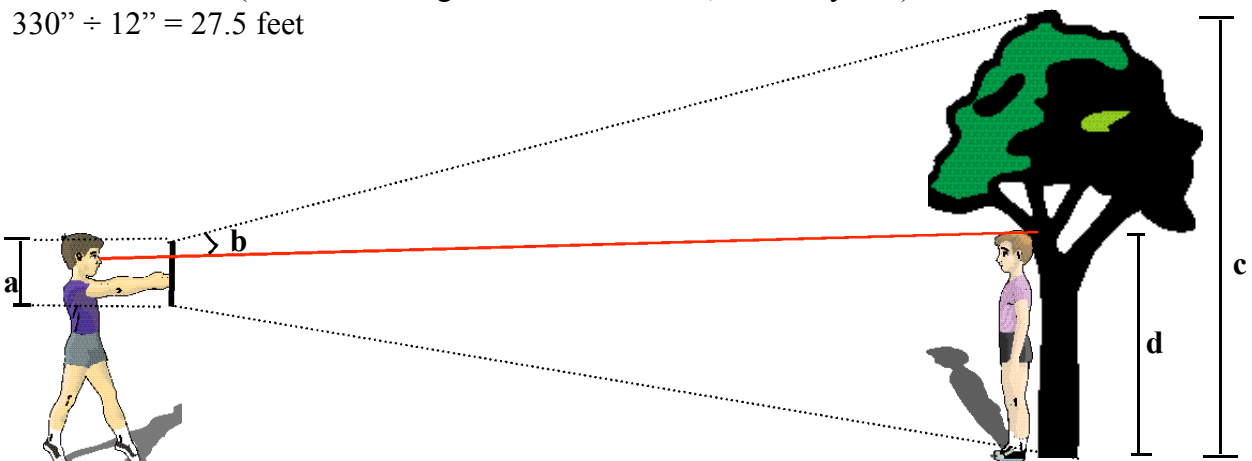
$$\text{Height of student} \times \text{proportional factor} = \text{Height of Tree}$$

(d) x (a/b) = (c)

EXAMPLE:

If your partner’s height was 55”, then the height of the tree would be

$55'' \times 6'' = 330''$. (To find the height of the tree in feet, divide by 12’)
 $330'' \div 12'' = 27.5$ feet



3. Estimating board feet of lumber in your tree.

The volume of wood in a standing tree may be estimated by obtaining two measurements (diameter and height) and applying these to a tree volume table. Volume of wood is usually measured in **board feet**. A board foot is a piece of lumber that is 12" x 12" x 1". One giant Sequoia could yield more than 500,000 board feet, enough to make 33 houses!

Use the tree volume table below to determine the number of board feet in your tree. **To use the chart, you must first divide the height of your tree by 16.** The quotient of the height divided by 16 may not be exactly a number listed on the chart so you will have to round to the nearest number to use this chart.

$$\frac{\text{tree height}}{16'} = \frac{27.5 \text{ ft}}{16'} = 1.71 \text{ (round to } 1 \frac{1}{2}\text{)}$$

When dividing the height by 16, you are estimating the number of 16 foot pieces of lumber you would get from the tree.

Example:

If your DBH is 10 and the number of 16' logs is 1 1/2, then your tree has a volume of 44 board feet.

DBH (inches)	Number of 16 foot logs in trees						
	1/2	1	1 1/2	2	2 1/2	3	3 1/2
	Volume in board feet						
10	21	34	44	55			
12	30	52	68	85	98		
14	42	74	99	124	143	162	
16	59	100	134	169	198	226	246
18	74	129	175	221	259	297	325
20	92	162	220	279	328	377	413

Using Proportions to Determine Tree Height

Ratios are useful ways to compare two quantities. When two ratios are equal to one another, it is called a proportion.

There are several ways to solve a proportion. One method is to cross multiply. You can solve a proportion by multiplying the numerator of the first fraction and the denominator of the second fraction and setting it equal to the product of the denominator of the first fraction and the numerator of the second fraction. Thus, for a proportion $\frac{a}{b} = \frac{c}{d}$ or $ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$

Our two equal ratios to determine tree height are as follows: $\frac{\text{Ruler Height of Tree}}{\text{Ruler Ht of Student}} = \frac{\text{Actual Tree Height}}{\text{Actual Student Ht}}$

The only thing unknown in this proportion is the tree's actual height. By substituting what is known in the appropriate places, we get the following equation:

$$\frac{12(\text{ruler ht of tree})}{2(\text{ruler ht student})} = \frac{c}{55(\text{actual student ht})}$$

Cross multiply and solve for c

$$2c = (12'')(55'')$$

$$2c = 660'' \quad c = 330''$$



Measuring Trees Worksheet

1. Record the DBH of your tree. _____

Remember, this measurement is taken 4.5 feet from the base of tree)

2. While holding the ruler at arm's length, what is the reading on the ruler where your partner's head is seen? This number is "b", the ruler height of student.

3. Figure the proportional factor. The formula is $\frac{a}{b}$ (*ruler height of tree*)
b (ruler ht of student)

4. Record your partner's actual height in inches _____

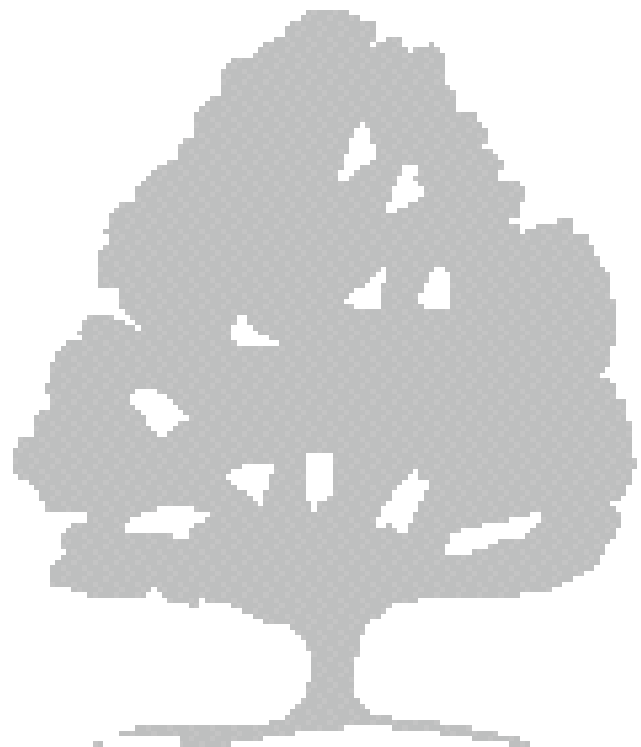
5. What is the height of your tree? _____ in inches _____ converted to feet

Height of tree = height of student x proportional factor

$$c = d \times \frac{a}{b}$$

6. Using your answer from #4, calculate the number of 16' boards found in your tree.
 (height of tree ÷ 16)

7. Using the tree volume table provided, find the number of board feet of wood present in your tree.



Patterns in Nature

Objective: Students will discover the Fibonacci sequence in nature and learn of the Golden Mean also known as Phi (pronounced Fi).

Background: Fibonacci was a medieval mathematician born in the year 1175 A.D. He is probably best known for a simple series of numbers introduced in his book, *Liber abaci*, and later named the **Fibonacci numbers** in his honor.

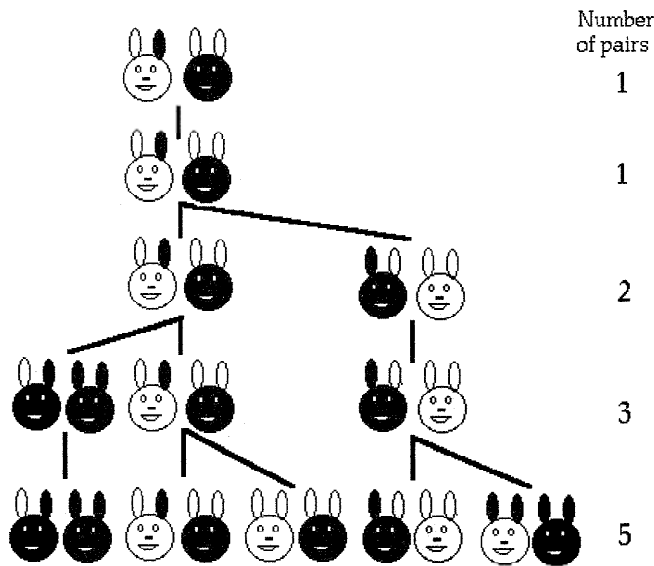


In Fibonacci's day, mathematical competitions and challenges were common and in one such competition, the following problem arose:

How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?

It is easy to see that 1 pair will be produced the first month, and 1 pair also the second month (since the new pair produced in the first month is not yet mature), and in the third month 2 pairs will be produced--one by the original pair and one by the pair which was produced in the first month. In the fourth month, 3 pairs will be produced, and in the fifth month, 5 pairs. After this, things expand rapidly and we get the following sequence of numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...



Do you see the pattern? To find the next number in the series, use the simple rule:

add the last two numbers to get the next

1,1,2,3,5,8,13,21,34,55,89,144,233,377,...

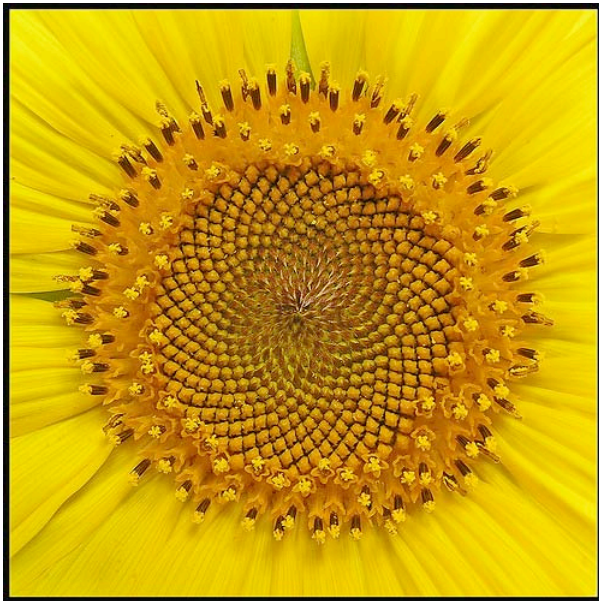
This simple, seemingly unremarkable recursive sequence has fascinated mathematicians for centuries. Its properties show up in an amazing array of topics, from the design of Greek buildings to growth patterns in plants.



Fibonacci Numbers in Nature

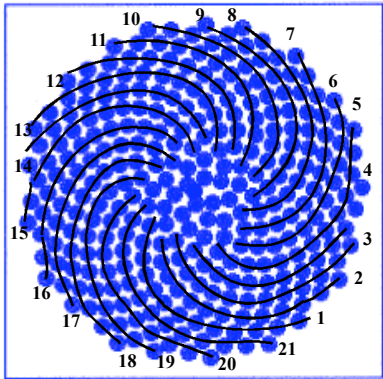
The rabbit breeding problem that caused Fibonacci to write about the sequence in *Liber abaci* may be unrealistic, but Fibonacci numbers really do appear in nature. For example, some plants branch in such a way that they always have a Fibonacci number of growing points. Flowers often have a Fibonacci number of petals--lilies or irises have 3 petals; buttercups, wild roses and larkspur have 5 petals; delphiniums have 8; ragwort has 13; asters and black-eyed susans have 21; and daisies may have 34, 55, or even as many as 89 petals.

Fibonacci numbers can also be seen in the arrangement of seeds on flower heads, such as that of a sunflower. The seeds appear to be spiralling outwards both to the left and the right. Guess what? The number of spirals to the left and the number of spirals to the right are both Fibonacci numbers. Broccoli and cauliflower, pine cones, and lettuce heads are other natural items that show Fibonacci spirals.

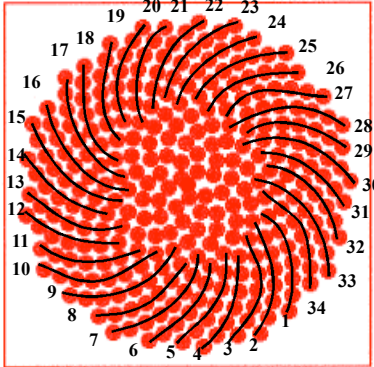


The seeds of a sunflower form two spirals, called parastichies, one set emanating from the center in a clockwise direction, the other in a counter-clockwise direction. In most cases, the number of clockwise spirals and the number of counter clockwise spirals will be adjacent numbers of the Fibonacci sequence.

The reason that many seeds seem to form spirals is that this arrangement allows the most seeds to be packed into the space. They are uniformly packed at any stage, all the seeds being the same size, no crowding in the center and not too sparse at the edges.



Left



Right



Patterns in Nature Worksheet

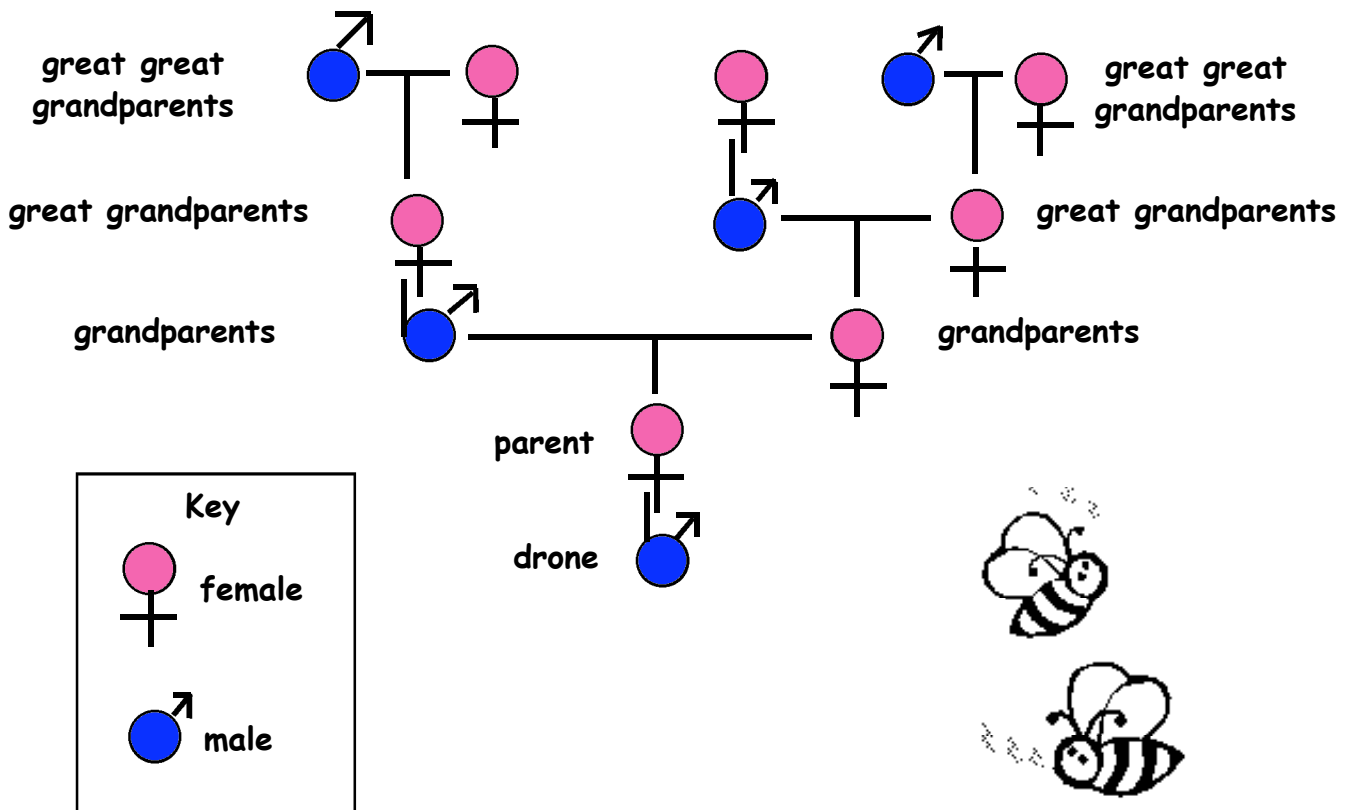
Part A Fibonacci Numbers

Honeybees and Family Trees. There are many kinds of bees but the one best known to us is the honeybee. It lives in a colony called a hive. There are many unusual features of the honeybee family tree such as:

- In a colony of honeybees, there is one special female called a **queen**. She produces **all** the offspring in the colony.
- There are many **worker** bees who are female but, unlike the queen, they do not produce eggs.
- There are some **drone** bees who are male and do no work. Males are produced by the queen's unfertilized eggs, so male bees only have a mother but no father!
- All the females are produced when the queen has mated with a male so have two parents.

Consequently, female bees have **2** parents whereas male bees have just **1** parent, a female. Let's look at the family tree of a male drone bee:

- He had **1** parent, a female.
- He has **2** grandparents, since his mother had two parents, a male and a female.
- He has **3** great-grandparents: his grandmother had two parents but his grandfather had only one.

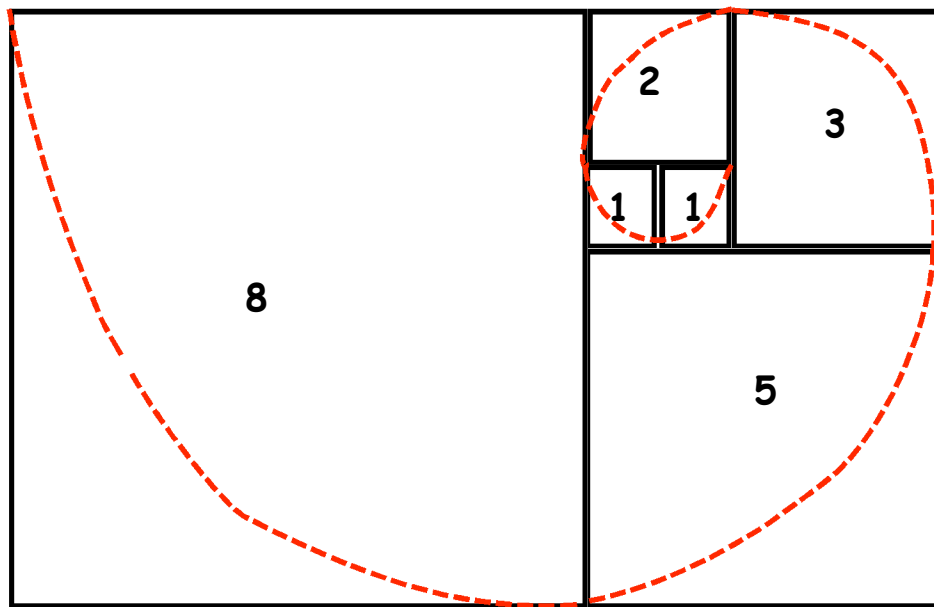


1. How many great-great-grandparents did the drone have? Fill in the chart below.

Number of:	Parents	Grandparents	Great Grandparents	Great, Great Grandparents	Great, Great, Great Grandparents
MALE bees	1	2	3		
FEMALE bees	2	3	5		

2. What is the pattern discovered in the family tree of honeybees?

3. We can show the Fibonacci numbers in another way. Start with two small squares of size 1 next to each other. On top of both of these, draw a square of size 2 (1 + 1). Next, draw a new square with one side touching the square of 2 and a square of 1--so having sides 3 units long; then another touching both a 2 square and a 3 square (so has sides of 5). Continue adding squares around the picture, **each new square having a side which is as long as the sum of the latest two square's sides**. This set of rectangles who sides are two successive Fibonacci numbers in length and which are composed of squares with sides which are Fibonacci numbers is called the **Fibonacci Rectangles**. Trace the spiral drawn in the squares (a quarter of a circle in each square).



While not a true mathematical spiral (like a spider's web), this is a good approximation of the kind of spirals that do appear often in nature. Such spirals are seen in the shape of the shells of snails or sea shells, in the arrangement of seeds on flowering plants (like the sunflower) and in the spirals of pinecones.

4. If the opportunity is available, gather pinecones from outside. Look at the base end where the stalk connects it to the tree. *Hint: Soaking the pinecones in water may make the spirals more visible.*

Can you see two sets of spirals? _____

How many are there in each set? _____

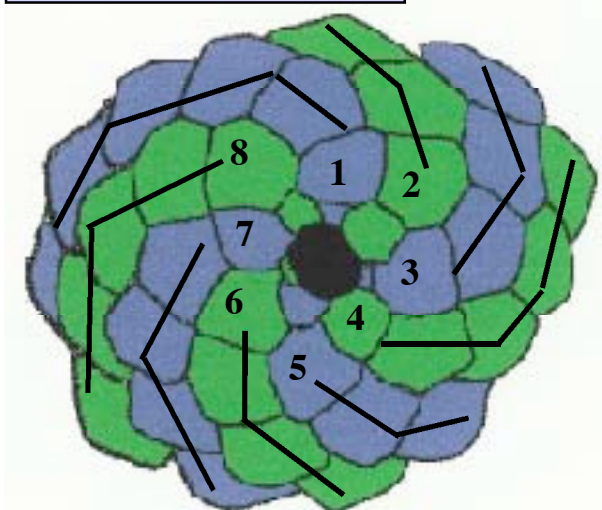
Are all the cones identical in that the steep spiral (the one with the most spiral arms) goes in the same direction? _____

Are the number of spirals Fibonacci numbers? _____

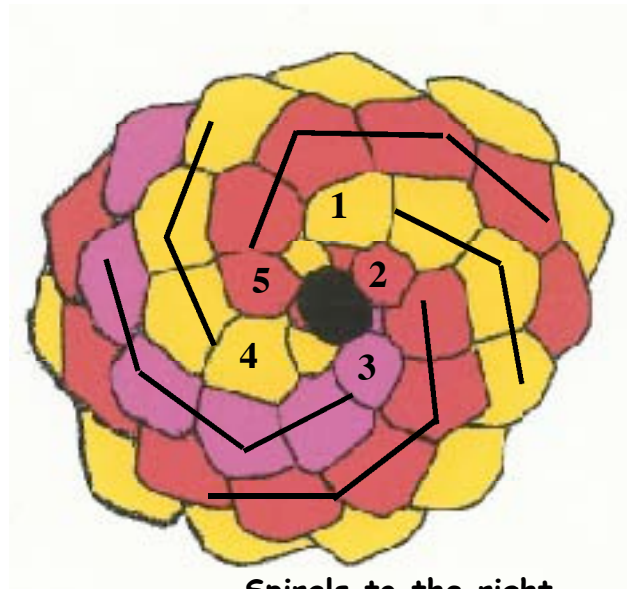


Pinecone spirals
Base end

Note: Pinecone spirals below are not of the picture above. Drawings from www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci



Spirals to the left



Spirals to the right

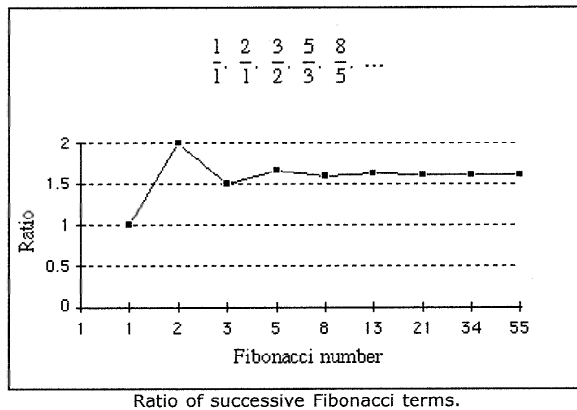


The Golden Mean

There is another value, closely related to the Fibonacci series, called the **Golden Mean**, or sometimes the Golden Ratio. This value is obtained by making a ratio of successive terms in the Fibonacci series:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \dots$$

If you plot a graph of these values, you will see that they seem to be tending to the same number--about 1.618. This number is denoted by the Greek letter *Phi*.



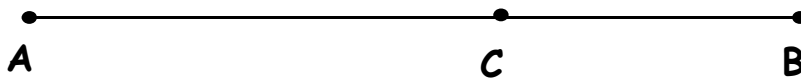
Golden Mean

$$\phi = 1.618$$

It seems, the Golden Mean represents an aesthetically pleasing proportion of height to width: 1 to 1.618. The ancient Greeks recognized the existence of the Golden Mean and used the ratio in the design of one of the world's most beautiful structures--the Parthenon.

Here is another way to look at the Golden Mean. The line below is divided according to the Golden Mean. This is one way the Greeks used the Golden Mean in their architecture to create pleasing lines.

When we divide line \overline{AB} at point C, we get the Golden Mean



$$\frac{AB}{AC} = \frac{AC}{BC} = 1.618033$$

The human body, as we shall see later, is also built around the Golden Mean.



Patterns in Nature Worksheet

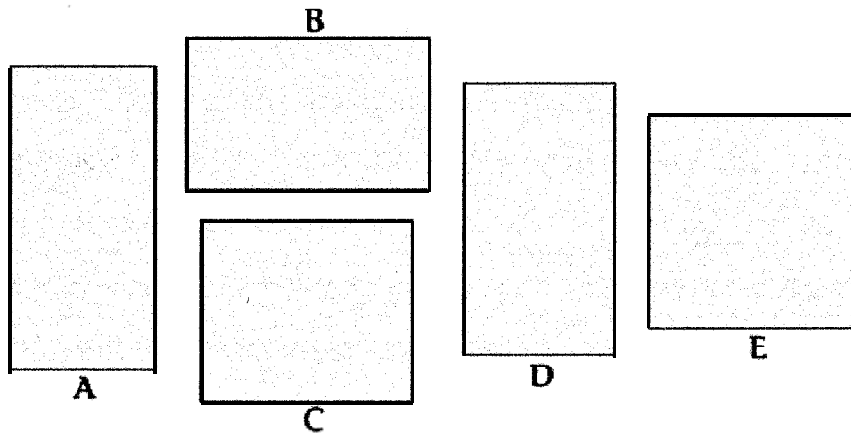
Part B The Golden Mean

Activity adapted from Ephraim Fithian at homepage.mac.com/efithian/Geometry. Used with permission.

Examine the five rectangles drawn below from several different views. Chose the one rectangle that is the most appealing to you and place an X on it. Use a metric ruler to measure both sides of each rectangle to the nearest millimeter. For each rectangle, divide the length of the longest side by the length of the shortest side and write this ratio on the rectangle.

1. Which rectangle was the most liked in the entire class? _____

2. What is its ratio? _____



3. Which rectangle is the Golden Mean? _____

4. Did you pick the Golden Mean as the most appealing? _____

5. What percentage of the class picked the rectangle showing the Golden Mean? (Hint: Divide the number picked by the class by the total number in the class, then multiply this decimal number by 100 for the percentage)



The Parthenon in Greece was built using the ratio of 1: *phi*





Beautiful Body

Do Fibonacci numbers and the Golden Mean appear in the human body?

Measure your height to the nearest centimeter and record it below. Measure the distance of your navel from the floor to the nearest centimeter and record that also. Divide your height by your navel height to find the height/navel ratio. Round this number to two decimal places and record the ratio below.

Height = _____ cm Navel Height= _____ cm Ratio $\frac{\text{(Total Height)}}{\text{(Navel Height)}}$ = _____ cm

6. Was your ratio 1.61? _____

7. What is the class average of these ratios? (Hint: Averages are found by dividing the total sum by the number of numbers in the set)

8. Was the class average closer to the Golden Mean than your results alone? _____

9. Measure the lengths of the bones in your index finger to the nearest millimeter (best seen by slightly bending the finger. You will have to feel for the 4th joint.)

1st joint _____ mm

2nd joint _____ mm

3rd joint _____ mm

4th joint _____ mm

Total Length _____ mm

a. Total Length \div 4th joint _____ mm

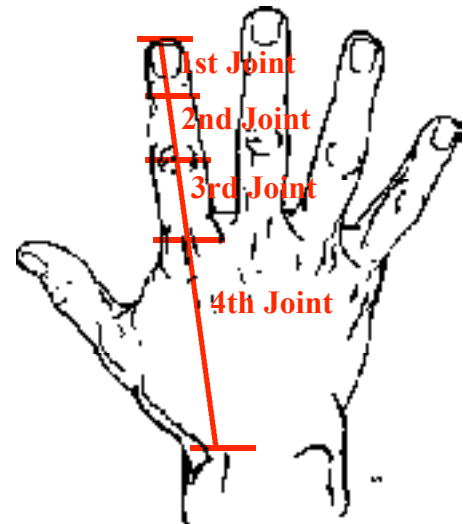
b. 4th joint \div 3rd joint _____ mm

c. 3rd joint \div 2nd joint _____ mm

d. 2nd joint \div 1st joint _____ mm

Average of a-d _____ mm

10. Did any quotients from **a-d** equal 1.61? _____



Is this coincidence? Curiously, you have 2 hands, each with 5 digits, and your 8 fingers are each comprised of 3 sections--all Fibonacci numbers!



11. Did your average come closer to 1.61 than the individual measurements? _____

12. Which hand did you measure? _____

13. Do you think both hands would be the same? _____

Can the Golden Mean Be Heard As Well As Seen?

(Activity adapted from www.enc.org/features/calendar)

The Golden Mean turned into a percent would equal 61.8 %. To find this number, we use only the decimal portion of *Phi* called *phi* with a small “p”. So *phi* is .61803 and *Phi* is 1.61803. To get 61.8%, we multiply *phi* by 100..

It seems that from Beethoven to Mozart or Faith Hill to Rascal Flats, at 61.8% of the way through a piece, something special almost always happens. It might be a sudden or brief change in key, a guitar or drum solo, bridge music (a musical transition between themes), a restatement of a theme, or even silence.

Let’s test the hypothesis that something often “happens” in a musical composition about 61.8 % of the way through it.

To Do: Provide several CD’s for the class to choose selections from.

14. Figure the number of seconds 61.8% through the piece. (Hint: To do this, we multiply the total number of seconds the song takes to play by .618)

Song A	Song B	Song C
Title:	Title:	Title:
Length of song (in seconds)	Length of song (in seconds)	Length of song (in seconds)
61.8% of song (in seconds)	61.8% of song (in seconds)	61.8% of song (in seconds)
Did “something” happen?	Did “something” happen?	Did “something” happen?

15. Do you agree with the statement that “something happens” 61.8% of the way through a musical composition?

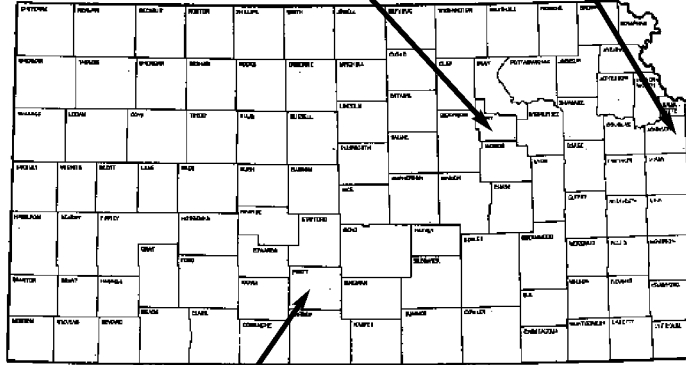


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